META-OPTIMISATION OF MIGRATION TOPOLOGIES IN
MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS

Abstract

Multi-objective evolutionary algorithms use an evolutionary approach to find Pareto-optimal solutions to problems with multiple competing objectives. One of the many classes of problem that they can be applied to is that of network optimisation – in particular the problem of finding an optimal topology of a network given certain criteria. For performance reasons, multi-objective algorithms are frequently implemented in parallel, using a number of distributed sub-populations. In this so-called island paradigm, migration is used to allow genetic material to move between sub-populations according to the migration topology.

This research combines the concepts of network optimisation and migration topologies, and examines the effectiveness of a multi-objective evolutionary algorithm in finding a Pareto-optimal migration topology for itself. The primary hypothesis is therefore that it is possible to find such Pareto-optimal topologies using the same algorithm at two levels. The results obtained support this hypothesis.

Background and aim of research

Multi-objective evolutionary algorithms (MOEAs) use a population-based approach and the principles of evolution to find Pareto-optimal solutions to problems with multiple competing objectives. Pareto-optimal solutions are ones in which these competing objectives are traded off against each other, and no solution can be said to be overall better than any other. The main concepts are illustrated in the following figure, reproduced from Abraham and Jain (2005).

Each solution has a fitness value for two objectives. With respect to solution \( x \), the top right quadrant shows solutions that dominate \( x \), because they have better fitness values for both objectives. Similarly, solutions in the bottom left quadrant are dominated by \( x \). Solutions in the top left and bottom right quadrants are neither dominated by, nor dominate \( x \), since they are better than \( x \) in one objective but worse in another. Finally, there is a set of Pareto-optimal solutions, which contains the solutions that are not dominated by any other solution. This is termed the non-dominated front.

Parallel distributed multi-objective evolution algorithms (PDMOEAs) are parallel versions of MOEAs that use a number of distributed sub-populations, with periodic migration of some individuals between the sub-populations to help maintain diversity in the overall population (Van Veldhuizen et al. (2002) and Lopez Jaimes and Coello Coello (2005)). The arrangement of the distributed sub-populations and the set of migration links between them is generally described as the topology of the PDMOEAs.

MOEAs are applied to many problems with multiple objectives, including network optimisation problems. One example in the literature is to find a network layout that minimises the average packet delivery delay, and minimises the network cost (Banerjee and Kumar (2007)).

This research combines all these ideas and investigates the use of a MOEA to obtaining an effective PDMOE topology in which the cost of migration is to be minimised, and the overall diversity of the final population is to be maximised. It represents an application of an evolutionary algorithm at two levels: the domain-level and the meta-level. The domain-level problem is the problem that is being solved by the PDMOE, and it may be any problem with multiple competing objectives. The meta-level problem is specifically the problem of finding an optimal migration topology for the PDMOE.
Research method

The main research method is empirical – the algorithm was simulated at a meta-level and a domain-level to test whether the results support the research hypothesis. Statistical tests were used to assign a confidence level to the conclusions. The following key tasks were required:

- Selection of a multi-objective evolutionary algorithm that was suitable for use at a domain-level and a meta-level. In this case, NSGA-II (Deb, 2000) was a practical choice with all the required characteristics. The algorithm was extended to support migration at the meta-level.
- Encoding of migration topologies using a suitable format. This was a binary encoding described in the dissertation.
- Selection of static topologies for comparison. This key analysis allowed fair comparison of the evolved solutions against a static solution to validate the results.
- Design of the appropriate objective functions to drive the meta-level fitness.

Objective functions

It is essential to have well defined objective functions at the meta-level to allow the algorithm to work. The diversity and cost metrics were used.

The cost of a topology is simply calculated from the sum of the weights in the graph. This is directly proportional to the total number of individuals migrated in each simulation run.

The diversity metric used is taken directly from Deb (2001) p. 328 and measures how well distributed the solutions in the first non-dominated front are. We call this the spread, and low spread values indicate more evenly spaced solutions. Two examples are shown in the graphs below. Each graph shows one non-dominated front. The axes are the two objective functions for the problem.

These concepts are illustrated in the figure below, which shows how the simulation operates. The working of the algorithm is shown as a “black box”.

![Graphs showing diversity and cost metrics](image-url)
Results

The main result of simulating some static topologies and some dynamically evolved topologies are presented here.

The left hand chart shows the initial random population of topologies. Each point is a topology, and each topology has a cost and a spread value, showing how diverse the domain level solutions are for that topology. The spread value has been found by simulating the PDMOEA defined by that topology. There are four non-dominated fronts.

The right hand chart shows the population after simulating the meta-level algorithm for 500 generations. Now there are three non-dominated fronts in the population. The solutions are all better than the random solutions in the initial population, because they have lower cost and lower (better) spread values.

Finally, the chart below combines the first non-dominated front from the final simulated population and the static topologies that we simulated separately.

From this chart we can clearly see that the evolved solutions are intermediate between the static solutions. For example, the connected graph has the highest cost, and produces high spread values – hence it is both expensive, and does not produce very diverse solutions. By comparison, the ring topology produces more diverse solutions than the evolved topologies and with a lower cost.
Conclusions

These key conclusions can be justified from the results.

- It is possible to evolve migration topologies. The migration topologies in the final population of the meta-level simulation had lower cost and mean spread values than the initial random solutions in the first population.
- The same multi-objective algorithm can be used at both a domain-level and at a meta-level, with only the necessary modifications to support migration. This links different areas of research together.
- The evolved solutions can be shown to be statistically different both from each other and from the original random solutions within a high level of confidence. This shows the use of robust statistical techniques to justify experimental conclusions.
- The evolved solutions are not better than all static solutions and this suggests further research into the role of migration and the necessary algorithms.

Future research

This research suggests some research areas worth investigating further. Some hypotheses are:

- There is a trade-off between diversity and convergence for MOEAs and rapid convergence may come at the expense of poor diversity. Can this be shown experimentally?
- There is no statistical difference between random migration and best-sent worst-replaced migration for MOEAs.
- Can a migration strategy be coupled directly to the fitness function to promote higher diversity for a particular algorithm (say NSGA-II)?
- A parsimonious graph representation would allow an evolutionary algorithm to avoid redundant evaluations of graphs that are isomorphic and significantly reduce simulation time.

References


